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Volatility Assessment of Pakistani Exchange Rupee Against World Renowned Currencies Using GARCH Models

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Abstract

Estimating exchange rate fluctuations is crucial for the smooth functioning of an economy, as it impacts both microeconomic and macroeconomic policies. Exchange rate movements can directly influence the prices of traded goods, as exemplified by China's currency valuation changes, which have had global repercussions. This paper evaluates the volatility of the Pakistani rupee (PKR) against major currencies—US dollar, Euro, Pound sterling, and Japanese yen—from November 1995 to October 2016 using data from "The University of British Columbia, Pacific Exchange Rate Service Database."To assess volatility, we applied autoregressive conditional heteroskedasticity (ARCH) and various generalized autoregressive conditional heteroskedasticity (GARCH) models, including Threshold GARCH (TGARCH), Exponential GARCH (EGARCH), and Power GARCH (PGARCH). The simple GARCH model was used to identify symmetric volatility effects, while TGARCH, EGARCH, and PGARCH models were employed to detect asymmetric effects through the inclusion of a leverage effect variable. Our findings indicate significant volatility in the PKR exchange rate against the benchmark currencies, with all lagged variance and residual coefficients being significant at the 1% level. Heteroskedasticity effects (ARCH) were noted in the residuals of the GARCH(1,1) model for the exchange rate against the yen. Consequently, we also forecasted improved models (GARCH(2,1), EGARCH(2,1), TGARCH(2,1), and PGARCH(2,1)) for the yen, Euro, US dollar, and Pound sterling. These models confirmed the significance of coefficients at the 1% level and validated the absence of ARCH effects in the residuals. The presence of leverage effects was evident in all three asymmetric models, implying that negative shocks (bad news) have a varying impact on future volatility, with higher effects seen for the Euro and Pound sterling compared to the US dollar and yen. The evidence of volatility clustering suggests that high volatility periods tend to persist, highlighting the importance of understanding these dynamics for economic stability.

Keywords: Volatility Assessment, GARCH Models, Pakistani Rupee, Exchange Rate, Asymmetric Effects

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Introduction

Important idea for Exchange rate flotation is it generates variety of effects on different units of economy, whereas it is not only the depreciation of rupee but overall volatility, which creates concern for the players in assets markets and general economy. Volatility creates sense of riskiness in the stock markets and effects the related agents thus the forecasting for currency rate is an attainable solution to pace down the speculative distortions. There had been notable improvements over the years based on empirical analysis and modeling criterion to bring volatility of currency into consideration specifically for the developed currencies and there are some very important applications of currency exchange rate modeling in private sectors due to its economic and financial importance for portfolio optimization and risk management, as pointed out by Ahmed E M (2011).

Notion of currency exchange rate is simply very close to the theory of money, via it plays an important role in the global market as a mode of exchange. While basic fundamental of money as store of value is shaken by the high volatility of currency, as speculative behaviors of its agents derives the price of currency above or below the expected. As the Streissler (2003) argues in his lecture "One of the most controversial issues in the international literature concerns the role of economic fundamentals in explaining exchange rate behavior...", while we see that exchange rate plays role into changing the value of capital and assets transactions, also the international commodities prices, many economists argue that exchange rate is relative prices of currencies whereas it contradicts the idea of non-neutrality of money, as empirical evaluation by working paper (Maurizio Michael Habib et al., 2016) published by European Central Bank suggests the significant effect of real exchange rate on the GDP growth rate. However, the natural price of currency should be gravitating to the long run price and it true value without inference from the government as any other normal commodity (Adam Smith, 1776). It creates an international dilemma, which has been a point of analysis for many economists; numerous models are taken under review to get clear insight for the estimation of currency like Pakistani rupee (PKR).

To counter this systemic issue of explosive volatility, there are different criterions being undertaken. One important idea for such analytics is similar to the study of Diebold and Nerlove (1989), who were working on the seven nominal dollar spot exchange rates, which found the evidence of Autoregressive Conditional Heteroskedasticity (ARCH) under multivariate model for the exchange rate. Which is also important implication of this model to be considered in the international assets markets since persistence or low volatility is consistent with the market efficiency, as these rates regulates demand and supply in correspondent to the market agents actions. Variance causes a greater change in the floatation of money and assets via different markets. Evidence of ARCH, as in for dollar currency, justifies the fact the volatility has a considerable and impactful driving effect from the exchange rate market to various other markets, in other words nonlinear serial dependence of currency exchange rate under multivariate conditions and inclusion of its past values can help to produce clearer prediction for the future currency rates, whereas such methodologies need to build a strong ground on the country by country basis before generalizing the results otherwise there will always be disagreement and uncertainty about the significance of this information.

It is established that volatility of currency's rate, has vast effects on the different macroeconomic variables of country such as in Pakistan. To the best of our knowledge, our estimation for the exchange rate of PKR against JPY, US and GBP using GARCH model have rarely been applied, whereas it has been used for the different dynamics explored in the paper of Jalil A K and Pervaiz, (2013) and also there has been recent work for India in the paper of Murari (2015), which produced important base work for our estimation. This criteria is also established on the fact that we are able estimate our volatility from the other financial units but is not in spectrum of our paper. Application of GARCH Model proved to be more efficient for the symmetric volatility in Indian Rupee (INR), while other branches of the model proved fruitful to capture the asymmetric dynamics in volatility of this currency. In consideration of changing global phases and increasing need of collaboration among the countries it is quite a necessity to build insightful criteria to understand the trend of currencies and as developing currency like PKR we see it as modifier for exchange rate market of Pakistan.

Literature Review

After the introduction of use of ARCH (Engle, 1982), these models had been used into numerous researches and empirical studies, basic purpose of such application is to forecast the volatility upon the prediction of various model. Simply, it is a method to evaluate any forecasting model, to help analyze and forecast volatility in trends. As the name suggest, its application revolves around the problem of heteroskedasticity of regression and such model includes the measure of non-constant variances in error based on the weighted average squared residual. ARCH Model has been generalized into different directions to accommodate different varieties of real world problems. One very useful extension is generalized autoregressive conditional heteroskedasticity (GARCH) (Bollerslev, 1986). This model considers average of past residuals and also internalize the effect of weightage, as the lags are increasing weights give to afar values of the variable goes nearer to zero. ARCH model is considerably easy for application and even in its simplest form, results are better estimate of predicting conditional variances. The most widely used GARCH specification asserts that the best predictor of the variance in the next period is a weighted average of the long run average variance, the variance predicted for this period and the new information for this period, which is the most recent squared residual. Various additions (Nelson 1990) & (Bougerol and Picard 1992) have been created during last decade which going accompany the estimation from GARCH.

GARCH model has their application to a wide range of time series analysis quantitative with major applications in finance and various quantitative analysis the assets and capital market. They have been particularly successful and have been point the focus for improvement, as in now large number of multivariate ARCH models to choose from Engle and Bollerslev et al., 1994. Numerous works have included modeling of conditional variance into practice is also one reason for its popularity in applied econometrics, (Nelson, 1991), (Bollerslev et al., 1992), (West and Cho, 1995), (Engle and Patton, 2001), (Evan and Lyons, 2002), (Shin, 2005) etc…, these publication has worked to estimate volatility of different financial markets. Recently, there is increase in the use of modeling tools to estimate the volatility, normally very speculative such as exchange rate market. There is increase in number of multivariate GARCH models to choose from, with the advancement in applied econometrics, challenges have gotten complex and require increasing applications.

We are going to discuss different extensions of GARCH but important idea is to build somewhat clear prescription for heteroskedasticity in the exchange rate of Pakistani rupee (PKR). Major role of heteroskedasticity and conditional variance is for accurate prediction of the variables after the shock in the past variables, (Rossi, 2013), regarding to issue of predictability of exchange rate her paper was confined to undertake the effects of different variables on the predictability of exchange rate but the assumption of linearity was one critical idea in use for her study, whereas such system may or may not be closely compatible for the prediction of the exchange rates, generally. We have taken a different approaches, which had been frequently used in estimation of conditional variance for the time series data, especially in stock markets such as in recent paper on Volatility Indian Stock Market by Harvinder Kaur (2004), this paper have notified many characteristics of the stock market from the emerging and developing economies.

Similar methods have been used to examine the variance in volatility of the exchange rate. Some local criterion for estimation of volatility as induced by the Zakaria & Abdalla (2012), trying to capture effects from various sources using both symmetric and asymmetric models that capture most common stylized facts such as volatility clustering and leveraging effect, through the application of EGARCH model. There finding for 19 Arab countries, implied that significant effect of the volatility on currency exchange rate for the majority of countries. Very important application of such finding comes out in terms of leveraging effect, as how very past shocks in rates effects heavily to the future rates of currency. Evidence of high volatility and leverage effect has also been modeled in study of Havery et al. (2008) for the estimation of effect of exchange rate volatility on Ghana's stock market, result of time series modeling shows that the negative relation relationship of volatility and returns, among two markets, however depreciation of currency leads to higher returns in the stock for developing countries, different scenarios explains different implications as of the developed countries (Kyle, 1993) where majority volatility of exchange rate, has effects of stock returns depending on the level of FDI and Domestic Units in different countries due which profit estimation fluctuate.

Looking into the previous literature, more general implication pave the path for the progress of the model which is being used in this paper. In fact, criteria for the paper is very similar as well as in line with major finding of the regional based countries. Major source volatility for our model captures symmetric and asymmetric volatility for Pakistani rupee, PKR against the renowned currencies US Dollar, Japanese Yen, Pound Sterling. There is an important tool used for estimation for conditional variance in the currency rate, GARCH model and it different extension going to give insightful results.

Background

Exchange rate measures the price of one currency in terms of another and can determine its competitiveness with its inverse relationship which means lower value of exchange rate means higher competition of that currency. Nominal exchange rate measures the relative price of two currencies for example USD against Pak Rupee, while real exchange rate indicates the relative prices of tradable goods in relation to non-tradable goods. The determination of exchange rate differs depending on the regime, in flexible regime it is determined by the interaction of supply and demand of exchange rates while under managed float crawling peg is used according to the change in prices of both home and foreign country.

According to the concept of purchasing power parity, a law of one price can applied on all goods and services if there are no transport and tariff costs between the countries. After Bretton-woods monetary collapsed in 1973, world major currencies started to float; due to this many countries pegged their currencies to dollar or French or other currencies.Then in the period of 1980s, developing nations began to adopt flexible exchange rate. Accordingly in 1996 less than 50% were pegged exchange rate while the percentage was 87% in 1975.

There were different justifications for the change in trend. High inflation in developing countries during 1980s led the countries in order to avoid loss of revenue depreciate their currencies against their trading partner. These led countries in Western Hemisphere to adopt a crawling peg, because then exchange rate could be adjusted according to the changes in inflation rate. The economic crises at that time were another reason for the move towards floating exchange rate. In 1980s, there was overall slowdown of industrial production growth, debt crises and surge in international interest rates. To cope up these challenges, not only discrete currency depreciation was required but also flexible exchange rate was necessary to counter those problems. Due to globalization, and trade openness, capital exchange has increased around the world thus bringing along the potential of shocks and emphasizing on more flexible exchange rate.

With many countries switching to floating exchange rate the volatility and uncertainty increased in economy and effects of exchange rate on different indicators varied accordingly. The starting of a regime of floating exchange rate contributed to economic liberalization but it also increased risks in doing international trade, because it created uncertainty of profits among exporters. The change in exchange rate can have significant impact on macroeconomic indicators such as prices, wages, unemployment, interest rates and level of outputs in the short run and it can affect the Gross Domestic Product, Foreign Direct Investment and Trade openness of the country in long run.It cannot be ruled out which regime of exchange rate was beneficial for overall macroeconomic performance. It is observed that inflation was least volatile during pegged exchange rate regime, while overall production has remained unchanged in both regimes.

Pakistan shifted to manage floating exchange rate in 1982, but the exchange rate partially behaved through free floating mechanisms. The rate was share-weighted float which means trade flows and bilateral currency fluctuations were allowed to affect the exchange rate value. The country adopted floating exchange rate on 17 July 2000. The automatic adjustment of exchange rate due to the free floating system have tendency to correct the balance of payment imbalances. Due to the imperfect market conditions and transmission mechanisms the changes in exchange rate do not correct the imbalances automatically which results into use of monetary and fiscal policy by government. The 9/11 caused the currency to appreciate against the dollar that earlier depreciated on the average of 1.5% each month. To contain depreciation initially discount rate and cash reserves requirement were used before September 2001, because there were many restrictions imposed on various transactions related to capital flows.

Data and Methodology

The time series data of Pak Rupee has been used against the most examined world currencies to test its volatility. Daily exchange rate of Rupee against dollar, pound sterling, and yen has been used to for the period November 1995 to October 2016. The Data has been collected from the most authentic source i.e The University of British Columbia, Pacific exchange rate service database”. The study includes daily returns which are calculated by taking first difference of logarithm of closing price of Pak Rupee exchange rate.

Volatility

As this thesis is based on assessing the volatility of exchange rate, it is important to define "Volatility" first. Volatility can be explained through standard deviation which includes the difference of the observed values from the mean. In Forex market, volatility is the risk or uncertainty involved in trading with the change in exchange rate, thus it affects the profitability of foreign exchange trades. It can be defined by spread of currency returns over some period of time. Lower volatility means exchange rate is concentrated over small values of range, while higher volatility tells its higher spread over large values in range. In many cases variance is also used to measure the volatility as in this paper. In floating exchange rates the value of currency is likely to fluctuate depending upon the strength of the economies involved in trading. As a result volatility might affect business dealing involving any two countries.

Stylized Facts about Volatility of Exchange Rates

Most variables of financial time series including exchange rate follow a certain stylized pattern which is necessary for model forecasting and estimation. Below are discussed some of most important stylized facts:

Heavy Tails

If we compare the return of foreign exchange rate with normal distribution, we observe heavy tails in terms of excess kurtosis. According to (Cont, 2001), for many normal distributions the standardized fourth moment is 3, while for financial time series it is found to be above.

Table 1: Descriptive Analysis

	PKR per Euro (RE)	**PKR per US Dollar (RD)**	**PKR per Pound Sterling (RP)**	**PKR per Japanese Yen (RY)**	**Log Difference of RE (LE)**	**Log Difference of RD (LRU)**	**Log Difference of RS (LRP)**	**Log Difference of RY (LRY)**
Mean	85.59679	70.03462	113.4901	0.674063	0.000173	0.000212	0.000172	0.000190
Median	77.29450	60.66550	112.9785	0.535205	8.69E-05	3.36E-05	0.000293	-4.78E-05
Maximum	148.8000	108.5510	177.9400	1.223300	0.297819	0.303778	0.297861	0.325076
Minimum	39.06400	34.17600	51.40200	0.304110	-0.301613	-0.303778	-0.300175	-0.301490
Std. Dev.	31.02816	21.13870	31.77742	0.266540	0.009864	0.007794	0.009617	0.010531
Skewness	0.204767	0.309573	-5.98E-05	0.462157	-0.019323	0.484269	-0.041449	1.344442
Kurtosis	1.554124	1.863681	1.981142	1.769271	333.9959	898.3718	366.6322	308.7461
Jarque-Bera	495.8799	367.7060	227.9434	520.2041	24052615	1.76E+08	29029644	20524487
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Sum	451095.1	369082.5	598092.8	3552.312	0.910082	1.118995	0.907042	0.999153
Sum Sq. Dev.	5072712.	2354425.	5320661.	374.3288	0.512545	0.319998	0.487239	0.584193
Observation	5270	5270	5270	5270	5269	5269		5269

Volatility Clustering

When values and changes tend to accumulate in the long run then this trend is known as volatility clustering. It is observed in most of series that values of different sizes in log-returns tend to cluster so when there is high volatility it will remain high and when there is low volatility it is to remain low. Volatility clustering is nothing but accumulation or clustering of information. (Engle and West 2005). It can be observed from figure 1 to 4, located below.

Leverage Effects

According to (Nelson, 1991), if we have negative correlation between volatility and asset return we call it leverage effect. There is decline in volatility if asset price rise and vice versa. It is observed in foreign exchange markets currency depreciation is followed by higher volatility.

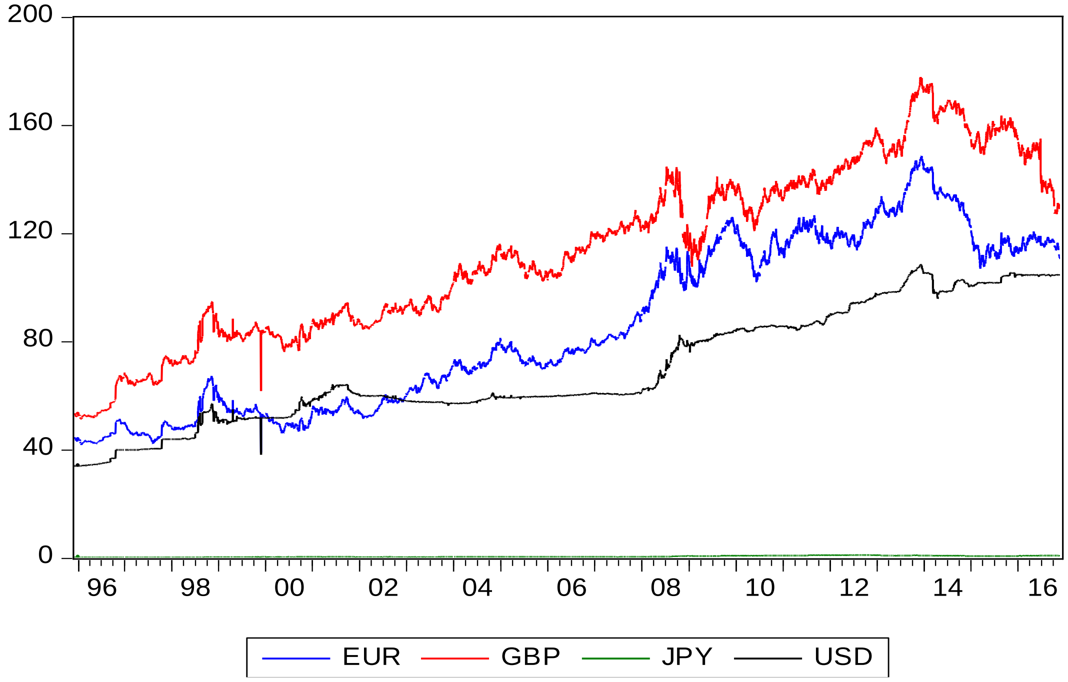
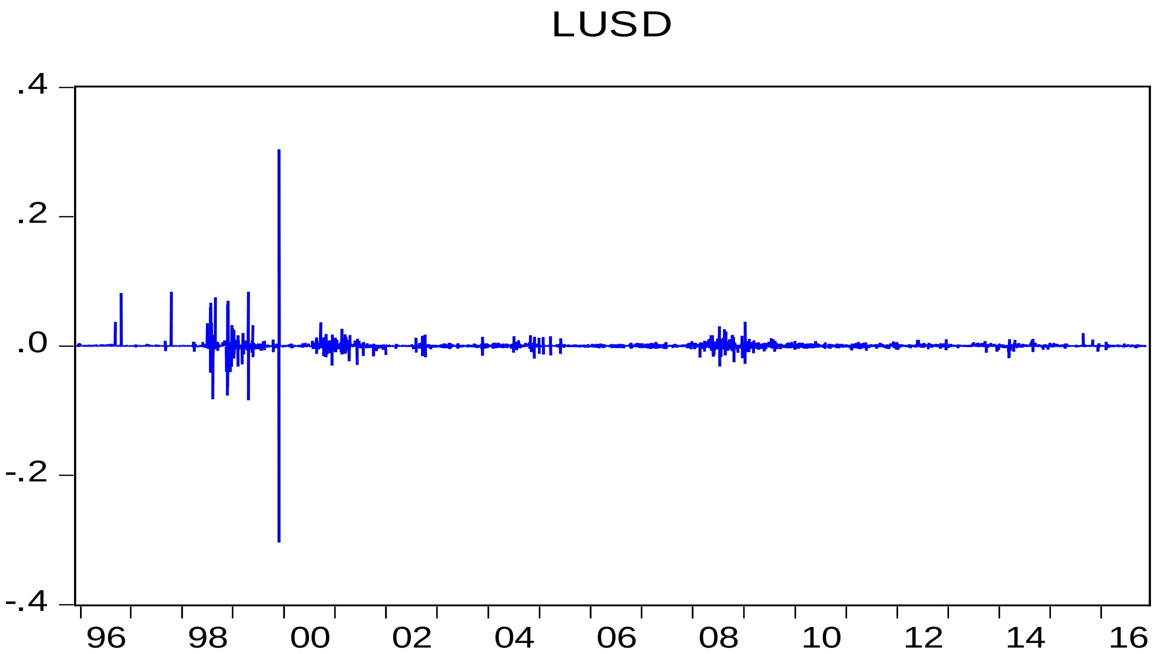


Figure 1: Volatility clustering of Pak Rupee against major currencies Leverage Effects



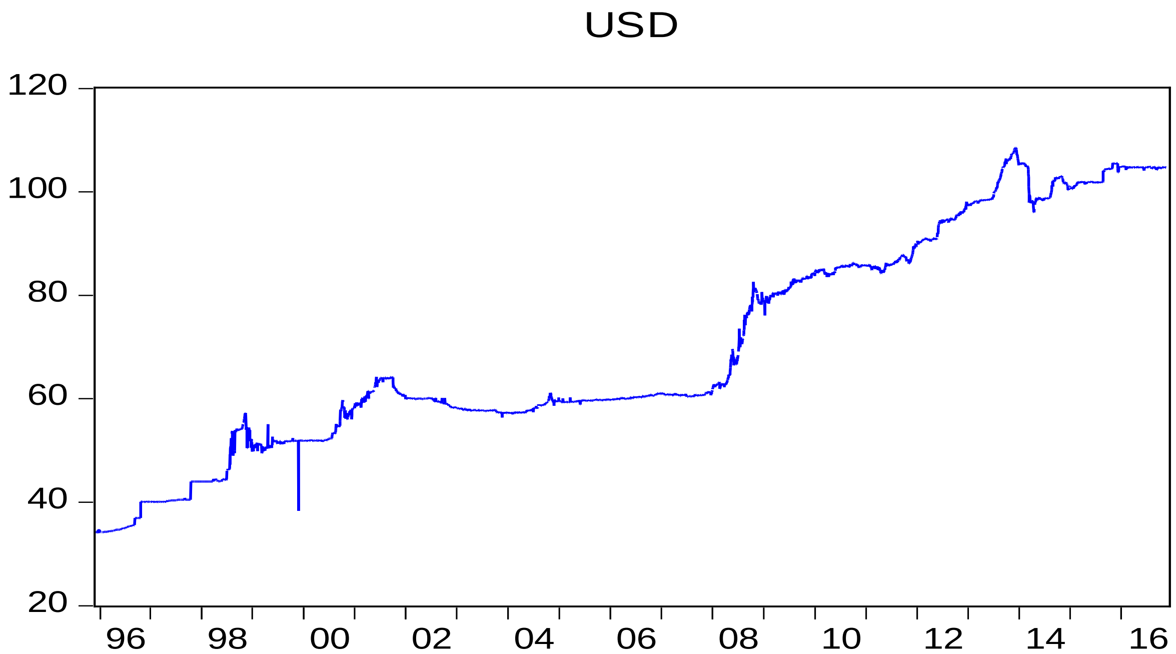


Figure 2: PKR per US Dollar (RU) and Log Difference of (RU)

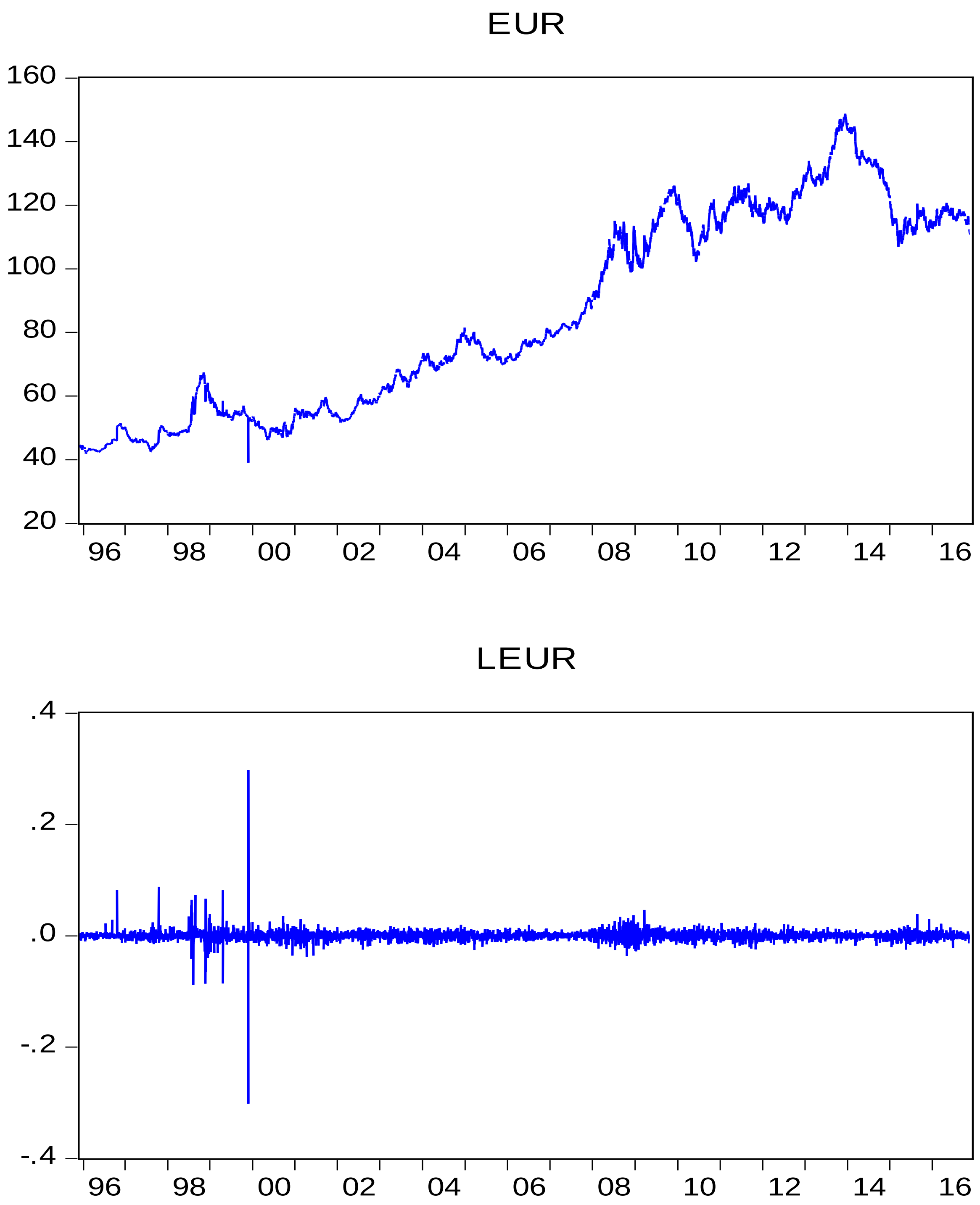


Figure 3: PKR to Euro (RE), and Log difference of (RE).

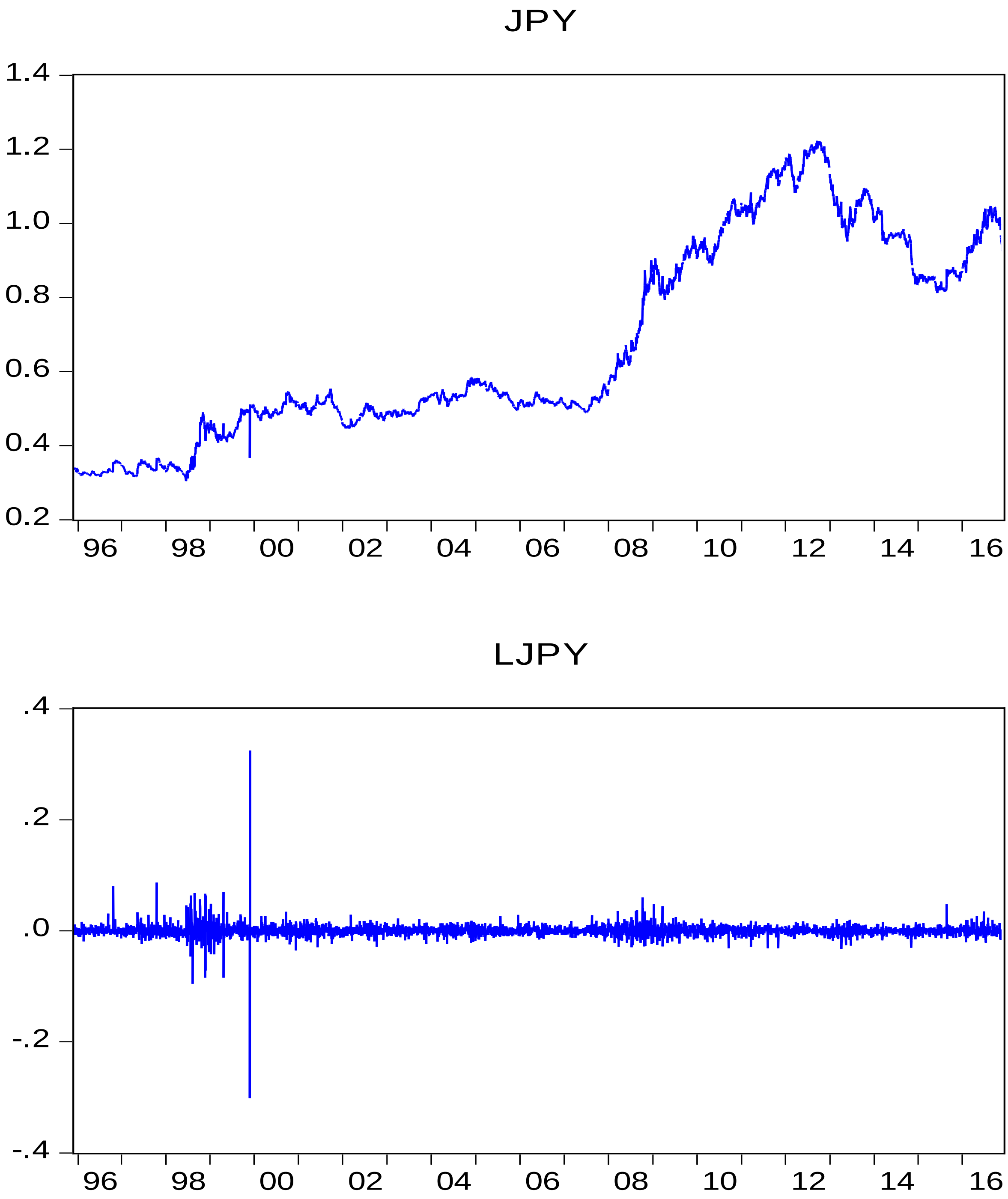


Figure 4: PKR to Japanese Yen (RY) and LOG difference of (RY)

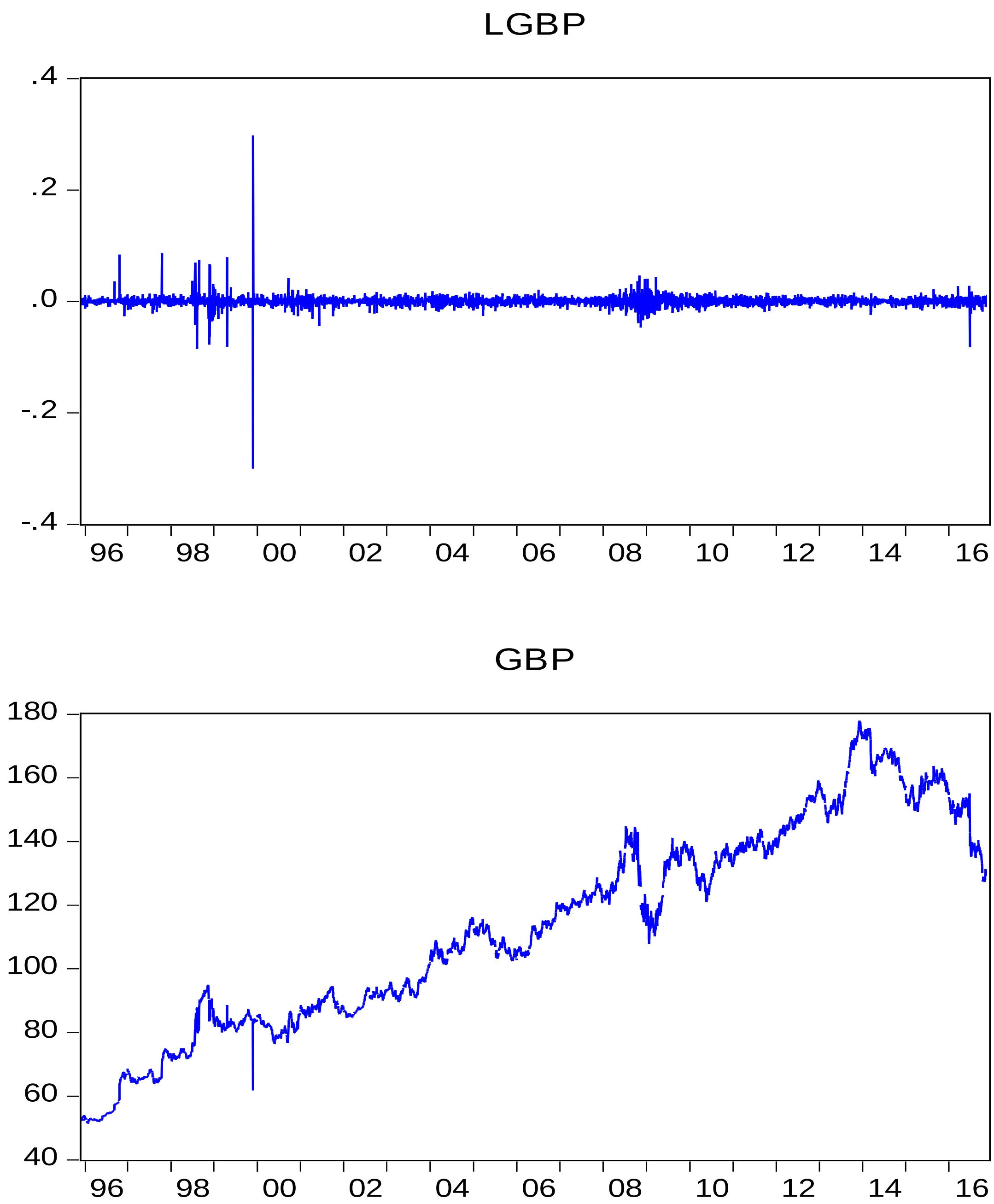


Figure 5: PRS TO Pound sterling (RP) and Log difference (RP)

Methodology

The linear time series models cannot capture major important attributes common to financial data. Mainly, volatility clustering that is described as, 'Large changes tend to follow by large changes, and small changes tend to follow by small changes...' Furthermore, annual deviation in price or value of financial assets is known as volatility.

The non-stationary condition of any series signifies the change in value over the time. It also implies conditional variances of the series change over the time. One model, which is used for such problem is ARCH model was presented by Engle (1982).

ARCH model's important idea evolves around the conditional and the unconditional second moments. While the unconditional covariance of the variables could be time invariant, the conditional variances and co variances often depend non-trivially on the past states of the world.

In underneath we present the model in the setting of predicting exchange rate changes. Contemplate the following preparation for the exchange rate return.

$$r_t = \mu + \varepsilon_t$$

r_t is the return at time, like day, t and ε_t is an independent observation from N (0, σ_t^2). Beneath a random walk supposition:

$$\sigma_t^2 = \sigma^2$$

For all t, i.e., the variance of ε_t is constant for all t. As well-known in past, the return r_t is homoskedastic. The variance of the exchange rate change does not track homoscedasticity, but show obligation on the variances of above periods. One method to model this serial correlation of the variance is:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 \qquad (1)$$

This construction is named as an Autoregressive Conditional Heteroskedastic (ARCH) model. The motive why it is entitled so is that ε_t^2 is a balanced approximation of σ_t^2 , i.e., $E\left(\varepsilon_t^2\right) = \sigma_t^2$, so that important the approximation error $v_t = \varepsilon_t^2 - \sigma_t^2$, we can re-write (1) for ε_t^2 as follows:

$$\varepsilon_t^2 = \sigma_t^2 + v_t = \omega + \alpha\varepsilon_{t-1}^2 + v_t \ (2)$$

AR(1) specification for the squared residual. Distinct the AR (1) model for the mean, it is the error term v_t is not continuous. Therefore (2) is a heteroskedasticAR (1).

We can simplify (2) to comprise squared residuals with lags 2, 3 and so on as descriptive variables. Subsequent the typical representation for AR models, it is appropriate to signify (2) as ARCH (1). Let *(L)* be a p-th order polynomial in lag operator *L*, i.e.,

$$\alpha\left(L\right) = \alpha_1 L + \ldots \alpha_p L^p$$

Then, ARCH (*p*) is:

$$\sigma_t^2 = \omega + \alpha\left(L\right)\varepsilon_t^2$$

As we know in ARMA modeling about the mean, it is not intelligent to keep aggregate lagged squared residual terms for explanation of the serial requirement of the variance. In its place, a convenient sparing simplification of (1) is to comprise σ_{t-1}^2 term as an explanatory variable, i.e.:

$$\sigma_t^2 = \alpha_0 + \alpha\,\varepsilon_{t-1}^2 + \beta\,\sigma_{t-1}^2 \qquad (3)$$

The simplification is recommended by Bollerslev(1986) and is termed GARCH-(Generalized ARCH). It can be revealed that (3) is a heteroskedastic ARMA (1,1) description for the squared residual. See below. From (3),

$$\varepsilon_t^2 = \sigma_t^2 + \nu_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \nu_t$$
$$= \omega + \alpha\varepsilon_{t-1}^2 + \beta\left(\varepsilon_{t-1}^2 - \nu_{(t-1)}\right) + \nu_t$$
$$= \omega + \alpha\varepsilon_{t-1}^2 + \beta\varepsilon_{t-1}^2 - \beta_1\nu_{(t-1)} + \nu_t$$
$$= \omega + (\alpha + \beta)\varepsilon_{t-1}^2 + \nu_t - \beta\,\nu_{(t-1)}$$

It is heteroskedastic, because ν_t is not constant. Following ARCH(1), (3) is denoted as GARCH(1,1).

Let $\beta(L)$ be a q-th order polynomial in L, i.e.

$$\beta(L) = \beta_1 L + \ldots \beta_q L^q$$

Then GARCH(p, q) is:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$$

The Threshold-GARCH (T-GARCH) model by Zakoian(1994) is comparable to GJR-GARCH. The description of conditional deviation in its place of conditional variance:

$$\sigma_t = K + \delta\sigma_{t-1} + \alpha_1^+\varepsilon_{t-1}^+ + \alpha_1^-\varepsilon_{t-1}^-$$
$$where\varepsilon_{t-1}^+ = \varepsilon_{t-1}^- if\,\varepsilon_{t-1}^- > 0, and\ \varepsilon_{t-1}^+ = 0\ if\,\varepsilon_{t-1}^+ \le 0$$
$$Likewise, \varepsilon_{t-1}^- = \varepsilon_{t-1}^+ if\,\varepsilon_{t-1}^+ \le 0, and\varepsilon_{t-1}^- = 0\ if\,\varepsilon_{t-1}^- > 0$$

The exponential-GARCH model by Nelson in 1991 is another different variety of the GARCH model. Properly, an EGARCH(p,q):

$$\log\sigma_t^2 = \omega + \sum_{k=1}^{q}\beta_k g(Z_{t-k}) + \sum_{k=1}^{p}\alpha_k log_{t-k}^{\sigma^2}$$

Where:

$g(Z_t) = \theta Z_t + \varphi(|Z_t| - E(|Z_t|))$, σ^2 is the conditional variance, $\omega, \beta, \alpha, \theta and \varphi$ are coefficients. Z_t Possibly will be a standard normal variable or originate from a generalized-error distribution. The devising for $g(Z_t)$ permits the signal and the scale of Z_t to have distinct effects on the unpredictability. EGARCH which has a number of advantages over the basic GARCH model, as the non-negativity constraint does not need to be imposed and the asymmetries are also allowed for using this model:

$$\ln(\sigma_t^2) = \chi + \beta\ln(\sigma_{t-1}^2) + \gamma\frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}]$$

Ding, Granger and Engle in 1993 also presented the Power-GARCH (PGARCH) description to pact with asymmetry. Contrasting additional GARCH models, in this model, the standard-deviation is slightly demonstrated as in contradiction of exhibiting of variance in most of the GARCH models. In Power-GARCH anon-compulsory parameter γ can be supplementary to explanation for asymmetry in demonstrating up to order r. The model also gives one the chance to approximation the power parameter δ instead of imposing it on the model Ocran and Bieketsin 2007. The over-all asymmetric Power GARCH model stipulates$\theta_t$as of the following form:

$$\sigma_t^{\delta} = \omega + \sum_{k=1}^{q} \beta_k \sigma_{t-1}^{\delta} + \sum_{k=1}^{q} \alpha_k (|\varepsilon_{t-k}| - \gamma_i \varepsilon_{t-1})^{\delta}$$

Results and Discussion

Detailed explanation of exchange rate of Pakistani rupee and its log difference against world stable economies US dollar, Pound sterling, Euro, and Japanese Yen, are described in table 1. The symbols (RE), (RD), (RP), and (RY) represent simple day to day exchange rate variable of Pakistani rupee against our world benchmark economies, while (LRE), (LRU), (LRP), and (LRY) variables show first log difference of day to day PKR against our selected economies. Our study period is from September, 1995 to October, 2016. During study period of almost 22 years, the minimum exchange values of rupee were39.06, 34.17, 51.40, and 0.304 against Euro, US dollar, British pound, and Japanese yen, whereas correspondingly, the maximum exchanges values were 148.80, 108.55, 177.94 and 1.22 against Euro, US dollar, British pound, and Japanese yen. Maximum variance in mean of exchange rate rupee values is against British pound, as standard deviations is highest 31.7742 against this currency. Now, approaching to the distributional properties, we know normal distribution has zero mean, while the positive mean distribution is rightly skewed, and negative distribution is skewed on the left side. In our study, the skewness for the log difference of exchange rate series LRE, LRU, LRP, and LRY are -0.019, 0.484, -0.041 and, 1.344, respectively. In the long run, positive and negative values of skewness, shows asymmetry. In our case, LRE and LRP shows negative values of skewness while LRU and LRY shows positive skewness, hence it shows data is asymmetric. There are three kinds of kurtosis in any kind of data, which are Meso, platy, and lepto kurtosis. Meso has same distribution as Normal with kurtosis measure coefficient (Moment coefficient of Kurtosis) 3. A value greater than (moment coefficient) Kurtosis 3, means flatness (Leptokurtic), while lesser than 3 means peakness (Platokurtic) of data series.

The kurtosis of all the series is different from standard value 3. Since kurtosis values of the series RE, RD, RP, and RY are 1.554124, 1.863681, 1.981142 and, 1.769271, which means lower than 3 and imply that data is flat and disperse. The higher the value of kurtosis for LRE, LRU, LRP, and LRY shows the data series is peaked, moreover whole series of first difference exchange rate is highly peaked with kurtosis 333, 898, 366 and 308, which is very high as compare to normal distribution. Jarque-Bera (JB) test of normality is also shown for all the data series, and it rejects the normality of data at 1% level of significance.

Figure 1 to 5 will give the visual inspection of the Pakistani rupee value and log differentiated against Euro, US dollar, British pound, and Japanese Yen. It is clear from pictures that frequent changes or high volatility in LRE, LRU, LRP and LRY data series is showing clustering.

Testing for Stationary of the Series

A time series data is called stationary if its mean, variance, and autocorrelation are constant over a period of time, or it can be explained as Hyndman and Athanasopoulos (2003) explained " stationary time series is one whose properties do not depend on the time at which the series is observed". There are many methods to find stationary of time series data, but two famous tests Augmented Dickey–Fuller (ADF) test (Dickey D A and Fuller W A, 1979)and Phillips–Perron (PP) test (Phillips P C and Perron P, 1988)have been applied in our study to estimate whether the daily rupee value against Euro, US dollar, Pound sterling, and JapaneseYen and their first log difference are stationary variables. The result of both unit root, DFand PP tests,on exchange rate variables RE, RU, RP and RY is shown in table 2, that show these variables are insignificant at 1% level of significance. In order to better forecast, next figure 3 shows the ADF and PP test statistics for log difference exchange rates, LRE, LRU, LRP and LRY are significant at 1% level, thus rejecting the null hypothesis of the presence of unit root in the data. Moreover LRE, LRU, LRP and LRY have DFT t-statistics values -60.44, 53.70, -60.75 and -59.40456, and PP t-test statistics values -93.13, -123.92, -94.54 and -92.23, which are highly significant at all levels so clearly rejecting the null hypothesis of unit root and confirming that that LRE, LRU, LRP and LRY are stationary at all levels.

Table 2: Unit Root Test

Stationary Test: Simple Variables

Table 2: unit root test				
Series	**ADF Test *t*-Statistics**	***p*-Value**	**PP test Adj. *t*-Statistics**	***p*-Value**
	(Null: Unit Root)		**(Null: Unit Root)**	
RE	-1.666066	0.7662	-1.677541	0.7613
RD	-1.567279	0.8058	-1.644257	0.7754
RP	-2.454221	0.3512	-2.351959	0.4050
RY	-1.358908	0.8726	-1.347797	0.8756

Table 3: First Difference Variable

Stationary –First Difference Variable

Table 3: unit root test				
Series	**ADF Test *t*-Statistics**	***p*-Value**	**PP test Adj. *t*-Statistics**	***p*-Value**

	(Null: Unit Root)		(Null: Unit Root)	
LRE	-60.44016	0.0001	-93.13886	0.0001
LRU	-53.70755	0.0001	-123.9237	0.0001
LRPS	-60.75287	0.0001	-94.54393	0.0001
LRY	-59.40456	0.0001	-92.23899	0.0001

Testing for Heteroskedasticity:

Residual is very important elements in estimating volatility. For assessment of volatility, first very importantly, we need to check whether residuals are homoskedastic or heteroskedastic, because one cannot use homoskedastic model to estimate volatility. Difference between both homo and heteroskedastic is that variance of the error terms is constant in homoskedastic models while it varies in heteroskedastic models. Thus, testing for the heteroskedasticity in residuals is necessary before modeling the volatility of Pakistani rupee exchange log return against Euro, US dollar, Pound, and Japanese Yen. First, we find error from Autoregressive and moving average (ARMA) process as shown by equation (A) below. The Autoregressive component of model tells that the present value of time series depends upon its previous values, while moving average component tells us it also depends upon its previous residual values. General form of the ARMA (p,q) model is as follows.

$$Y_t = \omega + \varnothing_1 Y_{t-1} + \pi_o \epsilon_t + \pi_1 \epsilon_{t-1} \qquad \text{(A)}$$

Where,

Y_t is the dependent variable at time t.

ω is the constant term.

π_0 and π_1 are the coefficients of the residual terms.

Y_{t-1} is the lagged dependent variable.

$\varnothing_1$ is the regression coefficients.

ϵ_t and ϵ_{t-1} are the current and previous value residuals.

ARCH-LM Test

In order to find heteroskedasticity in residuals of log exchange return rate series, Engle's Lagrange Multiplier (LM) test for ARCH effects (Engel, 1982) has been applied. Since the test is checking heteroskedasticity in errors, so first we derive relation from (A) in the residual form.

$$Y_t = \omega + \varnothing_1 Y_{t-1} + \pi_o \epsilon_t + \pi_1 \epsilon_{t-1} \text{ .... (A)}$$

Now residuals from (A)

$$\epsilon^2_t = \beta_o + \beta_1 \epsilon^2_{t-1} + \beta_1 \epsilon^2_{t-2} + \ldots\ldots\ldots \beta_p \epsilon^2_{t-q} + \nu_t \text{ ....(B)}$$

Since the residual equation is formed, now we can find heteroskedasticity in residuals of ARMA (1, 1) using ARCH-LM test.

Regression (B) is used to find ARCH LM test- statistics. Our null hypothesis to test is that there is no ARCH up to order p in the residual ϵ or i.e H_0: $\beta_o=\beta_1=\ldots\ldots=\beta_q=0$

Following table 4 shows the results of ARCH LM test to check the presence of autoregressive correlation hetrokedasticity in the residual series of LRE, LRU, LRP and LRY at lag three. If we see the ARCH statistics and LM-stats values of all the rupee exchange rate series, we can easily see large ARCH F-stats values and LM-statistics values, so this shows the series is very significant a 1% level. Also, p value is very lesser than 0.01 shows the presence of ARCH effect in the residuals of series thus we reject the null hypothesis of absence of arch effects in residuals of the series. Hence, it is obvious from the outcome of table 4 that there is a strong presence of heteroskedasticity (ARCH effect) in the residuals of log return series of Pakistani rupee against Euro, US dollar, British Pound and Japanese Yen. Therefore heteroskedasticity directs us to find volatility of exchange rate rupee using different type of GARCH models.

Table 4: ARCH-LM Test Results

ARCH-LM Test

Series	**ARCH *F*-Statistics**	**Prob. *F*(4,5259)**	**LM-Statistics**	**Prob□χ^2(3)**
LRE	376.8508	0.0000	1172.701	0.0000
LRU	275.5216	0.0000	714.7674	0.0000
LRP	506.1658	0.0000	1178.974	0.0000
LRY	774.4110	0.0000	1612.234	0.0000

Symmetric Model: GARCH (p, q)

GARCH model, usually, explains financial markets variance, in which data can vary and become more volatile during periods of financial crises and global events, both economic and politic-economic, or less volatile during periods of relatively calm and slow economic growth. On the global scale, before 2007 financial crisis, stock returns were relatively consistent. Meanwhile, following the onset of a crisis, returns may swing wildly from negative to positive territory. Moreover, the increased volatility may be extrapolative of volatility moving forward.

$$\sigma^2_t = \omega + \sum^{p}_{i=1}\alpha_i\epsilon^2_{t-1} + \sum^{q}_{i=1}\beta_i\sigma^2_{t-1}$$

GARCH processes, being autoregressive, model the current variance values by past residual squared observations and past variances. GARCH aims to minimize errors in forecasting by accounting for errors in prior forecasting of ARCH-LM, enhancing the accuracy of ongoing predictions.

The result of GARCH (p, q) model estimating the daily Pak rupee exchange rate volatility against Euro, US dollar, Pound sterling, and Japanese Yen for the sample period ranging from October, 1995 to September, 2016 has been shown in following table 5.

Table 5 shows all the coefficients of rupee against our benchmark currencies, ω (constant), α (Arch effect) and β (GARCH effect), in the sample period of 22 years which are statistically significant at 1% level as marked by the *. Only GARCH (1, 1) for LRY shows the presence of further ARCH in the residuals as F-statistics is 55 with P value less than 0.05 that shows heteroskedasticity in residuals. Thus we estimate GARCH (2, 1) model for LRY necessarily and for others to estimate more effectively, which show significant ARCH-LM test statistics for LRY, and further improved for LRE, LRU and LRP. The lagged conditional and squared variances have impact on the volatility and it is supported by significant ARCH terms (α) and GARCH terms (β).

Table 5: Estimation Results of GARCH (p, q) Model

	LRE		**LRU**			**LRP**	**LRY**	
	GARCH (1,1)	**GARCH (2,1)**	**GARCH (1,1)**	**GARCH (2,1)**	**GARCH (1,1)**	**GARCH (2,1)**	**GARCH (1,1)**	**GARCH (2,1)**
Constant(ω)	2.72E-05	6.01E-08	2.40E-05	6.81E-06	3.20E-05	1.89E-06	5.79E-08	2.18E-05
ARCH Effect (α_1)	0.632222*	0.124029*	0.225199*	0.175	0.872728*	0.837995*	0.012697*	0.127425*
ARCH Effect (α_2)		- 0.115802*		- 0.1130		- 0.751129*		0.521760*
GARCH Effect (β_1)	0.293743*	0.991800*	0.426680*	0.823749	0.110066*	0.923306*	0.988200*	0.368669*
($\alpha_i + \beta_j$)	0.9360	1.000027	o.6518	0.9012	0.9998	1.0101	0.9999	1.001
Residual Diagnostics: ARCH-LM Test								
F-Statistic	0.008612	0.220048	0.004867	0.010150	0.001855	4.24E-05	55.79504	0.240067
Prob. *F*(1,5259)	0.9261	0.6390	0.9444	0.9198	0.9656	0.9948	0.0000	0.6242
Note: * indicates that the coefficients are significant at 1% level.								

The coefficient α (ARCH effect) and β (GARCH effect) shows high statistical significance for rupee exchange rate against our World benchmark economies. Hence, by table 5 results, we can conclude that the variance of exchange Pakistani rupee (Volatility) is significantly affected by pervious variances and residual terms, so volatility is proved by symmetric GARCH model.

Furthermore, in previous table 5, the sum of ARCH and GARCH coefficients terms, i.e. residuals and previous variance coefficients α and β (persistent coefficients), in GARCH (1,1) model are near one for LRE, LRP and LRY, which is suggesting that the shocks to conditional variance are highly persistent or in other words the conditional variance process is volatile. GRACH and ARCH coefficient sum for the log difference of Pakistani exchange rate against series LRE, LRP AND LRE, in GARCH (2, 1), are nearly one, which means volatility clustering occurred and persisted in the long run. Only GARCH (1, 1) model for log difference exchange rate of Pakistani rupee again US dollar (LRU) has sum of coefficients α and β which is lower than 1 which implies shocks of exchange rate of Pakistan rupees by US dollar are not very long lasting, though the sum is greater than 0.65, hence it is greater than the value in the short run, but lower than the long run as evidenced from figure 7.

Asymmetric GARCH Models

Threshold GARCH/TGARCH (p, q):

General specification for the conditional variance under TGARCH (Glosten I R, Jagannathan R and Runkle , 1993) is given by:

$$\boldsymbol{\sigma^2_t = \omega + \sum^{p}_{i=1}\alpha_i\epsilon^2_{t-1} + \sum^{q}_{i=1}\beta_i\sigma^2_{t-1} + \sum^{r}_{k=1}\gamma_k\,\epsilon^2_{t-k}I_{t-k}}$$

Where I_t=1 if $\epsilon_t < 0$ and 0 otherwise.

In this model, good news is $\boldsymbol{\epsilon_{t-i} > 0}$, and bed news is $\boldsymbol{\epsilon_{t-i} < 0}$, have different effect on the conditional variance. Good news has an impact of $\boldsymbol{\alpha_i}$, while bed news has an impact of $\boldsymbol{\alpha_i+\gamma_i}$. If $\boldsymbol{\gamma_i}$ >0, it means increase volatility due to bad news, and it is believed that there is a leverage effect for the i^{th} order. If $\boldsymbol{\gamma_i \neq 0}$ and leverage term is significant, then asymmetric absence cannot be accepted.

Basically to test leverage or asymmetric effect, in the daily foreign exchange rate volatility of Pakistani rupee against Euro, US dollar, Pound sterling and Japanese Yen, the TGARCH model has been used. Table 6 shows the predicted result of TGARCH (1, 1) and TGARCH(2, 1) for the series LRE, LRU, LRP and LRY and the resultant coefficients are significant mostly at 1% confidence level, except for TGARCH(1, 1) leverage coefficient (Gamma) which is significant at 5% level of confidence. Now, coming towards asymmetric analysis of TGARCH (p, q), we analyze negative shocks i.e. leverage term (γ). Statistically significance of leverage terms at 1% level confidence suggests that there exist a leverage effect or asymmetric behavior in daily Pakistani rupee exchange rate against Euro, US dollar, pound sterling and Japanese Yen. Moreover for LRU and LRY leverage term shows negative sign, indicating the negative shocks have lower effect on the future volatility. On the other hand leverage coefficients for LRE and LRY show positive sign, indicating the negative shock (bad news) have higher effects on future volatility as compared to other currencies. TGARCH (1, 1) model for the series does not have ARCH effect in its residual, so it means all equations are well precise. We have also found TGARCH (2, 1) in order to see better estimates of the model, and it is evident from figure 6 that all the variables are significant including leverage effects at 1% level of significance.

Table 6: Estimation Results of TGARCH(p, q) Model

	LRE		**LRU**		**LRP**		**LRY**	
	TGARCH (1,1)	**TGARCH (2,1)**	**TGARCH (1,1)**	**TGARCH (2,1)**	**TGARCH (1,1)**	**TGARCH (2,1)**	**TGARCH (1,1)**	**TGARCH (2,1)**
Constant(ω)	2.67E-05	2.87E-05	2.39E-05	3.78E-05	8.27E-05	8.11E-05	1.29E-05	1.97E-05
ARCH Effect (α_1)	0.236962*	0.211735*	0.249904*	0.154998*	0.305212*	0.302335*	0.587906*	0.198869*

ARCH Effect (α_2)		0.033223*		-0.095608*		0.019048**		0.436638*
Leverage Effect (γ)	0.656896*	0.700302*	-0.030425**	0.127128*	1.104968*	1.118344*	-0.502430*	-0.110931*
GARCH Effect (β_1)	0.326262*	0.282608*	0.417684*	0.511885*	0.073667*	0.062382*	0.641104*	0.421311*
($\alpha_i + \beta_j$)	0.56322	0.5275	0.6674	0.5712	0.378	o.3833	1.2290	1.056
Residual Diagnostics: ARCH-LM Test								
F-Statistic	0.015119	0.013254	0.004674	0.019630	0.006686	0.006520	0.407607	0.463176
Prob. *F*(1,5259)	0.9021	0.9083	0.9455	0.8886	0.9348	0.9356	0.5232	0.4962
Note: * and ** indicate that the coefficients are significant at 1% and 5% levels, respectively.								

Exponential GARCH/EGARCH(p, q) Model:

(Nelson, 1991) Introduced EGARCH (p, q) model's variance equation is as follows:

$$\log(\sigma_t^2) = \omega + \alpha(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}) + \beta\left[(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}}) - \sqrt{\frac{2}{\pi}}\right] + \gamma \log(\sigma_{t-1}^2)$$

In exponential GARCH model, the asymmetric response of variance to shocks is captured and at the same time the positive variance is also ensured. In this model, leverage effect can be checked by hypothesis $\gamma_i < 0$. If γ_i is not equal to zero then that is asymmetric effect.

Following table 7 shows the results of EGARCH (p, q). The coefficients of all the series LRE, LRU, LRP and LRY are significant for both EGARCH (1, 1) and EGARCH (2, 1) at 1% and 5% level of significance, except for leverage coefficient of LRP in EGARCH (1,1) model. Though, Leverage coefficient of LRP in EGARCH (2, 1) is significant at 1% level of confidence.

The significance of leverage coefficients for EGARCH (p, q) shows the asymmetric behavior of volatility of Pakistani rupee against Euro, US dollar, Pound sterling and Japanese Yen. Positive Leverage coefficient of Pakistani rupee against US dollar in EGARCH (1, 1) shows that positive shocks (good news) have more effect on volatility as compare to the negative shocks (Bad news) have on the other three exchange rates LRE, LRP and LRY. Lastly, under EGARCH (1,1) the null hypothesis of no presence of heteroskedasticity in error of LRE, LRP and LRY is rejected as ARMA-LM test has shown the ARCH effects in the residuals of above series, while only exchange rate of Pakistani rupee against US dollar's residuals under EGARCH (1,1) has no presence heteroskedesticity in its residuals. To eliminate heteroskedasticity in residuals, better model EGARCH (2, 1) has been shown for all the series, and that has eliminated the effects of ARCH effects in residuals of all series, and all the equations are well precise.

In EGARCH (1, 1) and EGARCH(2, 1), sum of ARCH and GARCH coefficient is nearly equal to one, suggesting that the conditional variance shocks are highly persistent, in other words conditional variance is volatile.

Table 7: Estimation Results of EGARCH (p, q) Model

Estimation Results of EGARCH(p, q) Model								
	LRE		**LRU**		**LRPS**		**LRY**	
	EGARCH (1,1)	**EGARCH (2,1)**	**EGARCH (1,1)**	**EGARCH (2,1)**	**EGARCH (1,1)**	**EGARCH (2,1)**	**EGARCH (1,1)**	**EGARCH (2,1)**
Constant(ω)	-0.048409	-0.354706	-8.633090	-12.32007	-0.011091	-0.035315	0.005774	-2.353829
ARCH Effect (α_1)	0.045297*	0.328496*	0.930734*	1.145180*	0.031212*	0.400973*	0.043900*	0.320132*
ARCH Effect (α_2)		-0.194471*		0.432437*		-0.375160*		0.183376*
Leverage Effect (γ)	-0.028676*	-0.075699*	0.469015*	0.553549*	-0.000270	0.042141*	-0.009146*	0.143902*
GARCH Effect (β_1)	0.997832*	0.972053*	0.178290*	-0.148273*	1.000686*	0.997975*	1.003640*	0.784944*
($\alpha_i + \beta_j$)	1.043129	1.1057	1.1089	1.725	1.0312	1.0649	1.0470	1.285
Residual Diagnostics: ARCH-LM Test								
F-Statistic	29.56577	0.001979	0.001262	0.001443	313.0473	0.003238	254.1180	0.097798
Prob. *F*(1,5259)	0.0000	0.9645	0.9717	0.9697	0.0000	0.9546	0.0000	0.7545
Note: *,, and *** indicate that the coefficients are significant at 1% , 5% and, 10% levels, respectively.**								

Power GARCH/ PGARCH (p, q) Model:

(Taylor, 1987)Established the standard deviation GARCH model, where instead of variance, standard deviation is used.

$$\sigma_t^{\delta} = \omega + \alpha\left(\left|\varepsilon_{t-1}\right| - \gamma\varepsilon_{t-1}\right)^{\delta} + \beta\sigma_{t-1}^{\delta}$$

Power GARCH model is also called standard deviation GARCH model, because instead of modeling for variance, the standard deviation is modeled. The Power constraint δ of the standard deviation can be projected and the optional γ are added to capture irregularity up to some order r.

Table 8 shows below the result of standard deviation modeling (PGARCH). It is evident from table that all the coefficients of PGARCH (1, 1) are significant at 1% and 5% level of significance. Also, all the coefficients of leverage effect, irregularity capture, are significant at 1% level so it means irregularity or volatility is observed. Since, LRU is showing negative shocks (Bad news) which is associated with higher volatility as compare to lower volatility of positive shocks(good news) in LRE, LRP and LRY. The ARCH-LM test statistics exhibit ARCH effect in the residuals for LRY under PGARCH (1, 1) model, and does not exhibit any ARCH effect in residuals for LRE, LRU and LRP. Though, we have perform PGARCH (2, 1) for the all the series for better estimation, the ARCH-LM test not only accepting hypothesis of presence of no ARCH effect in residual of LRE, LRU and LRP, but also accepting for LRY. Hence, that is evident of well specified variance equation.

Estimation Results of PGARCH(p, q) Model

	LRE		LRU		LRPS		LRY	
	PGARCH (1,1)	**PGARCH (2,1)**	**PGARCH (1,1)**	**PGARCH (2,1)**	**PGARCH (1,1)**	**PGARCH (2,1)**	**PGARCH (1,1)**	**PGARH (2,1)**
Constant(ω)	0.000191	0.000580	6.66E-07	5.08E-05	1.98E-07	4.88E-07	-4.78E-07	2.57E-06
ARCH Effect (α_1)	0.111131*	0.267061*	0.349856*	0.166309*	0.920711*	0.334787*	0.017934*	0.164650*
ARCH Effect (α_2)		-0.210533*		-0.079833*		-0.326314*		-0.079833*
Leverage Effect (γ)	0.609246*	0.128838*	-0.044233***	0.139717**	0.322579*	-0.021023*	-0.015408***	-0.055271*
GARCH Effect (β_1)	0.897886*	0.946865*	0.367312*	0.508968*	0.037658*	0.993542*	0.988417*	0.991546*
Power Parameter	1.076071*	0.665683*	2.650752*	2.003048*	3.047707*	1.627074*	1.496258*	1.274632*
$(\alpha_i + \beta_j)J$	1.0100	1.0567	0.7125	0.77325	0.9756	1.0012	1.01356	1.0762
Residual Diagnostics: ARCH-LM Test								
F-Statistic	1.75791	0.935884	9.89E-06	0.091984	0.014605	0.007613	183.3002	2.658132
Prob. *F*(1,5259)	0.1849	0.3334	0.9975	0.7617	0.9038	0.9305	0.0000	0.1031

Note: *, **, and *** indicate that the coefficients are significant at 1%, 5% and, 10% levels, respectively.

Conclusion

In economics, for formation of public policies, trade policies, and even politic-economic policies, one of the most important factors considered nowadays is exchange rate volatility. In finance, exchange rate volatility is used in many financial applications like portfolio management, asset pricing and risk analysis. Our thesis, by using times series econometric and statistical models, is an attempt to investigate volatility of Pakistani rupee exchange rate against world renowned currencies Euro, US dollar, Japanese yen and pound sterling. Symmetric and conditional asymmetric applications of GARCH models to capture the most stylized facts such as volatility clustering and leverage effect on rupee exchange rate are examined.

Initially, GARCH (1, 1), TGARCH (1, 1), EGARCH (1,1) and PGARCH (1,1) models were used for log difference of rupee exchange rate against Euro, US dollar, Pound sterling and Japanese yen. The ARCH effect is present in the residuals of LRY in GARCH(1,1), EGARCH(1,1) and PGARCH(1,1). Also it is present in the residuals of LRE and LEP in EGARCH(1, 1). For the better estimation of volatility, two residual lagged models GARCH(2,1), TGARCH(2,1), EGARCH(2,1) and PGARCH(2,1) are used in the absence of ARCH effects in residual of LRE, LRP and LRY.

Fundamental rationale of GARCH (1, 1) and GARCH (2, 1) models was to capture conditional movement of variance, whereas TGARCH, EGARCH and LGARCH are used to capture asymmetric effects anticipating leverage effects within them. At 1% significance level, most of the coefficient of all the models found significant and thus proves that all the models provide strong evidence of volatility of daily exchange rupee return against Euro, UD dollar, Pound sterling and Japanese yen. Coefficients of leverage effect, in the asymmetric models, were non-zero ($\gamma \neq 0$) and significant at 1% level, so it means hypothesis of presence of asymmetry has been observed. Negative shocks have fewer effects on future volatility in case of US dollar and Japanese yen. And negative shocks, in case of Euro and Pound sterling, have higher effect on future volatility as shown from figure 6 to 9.

Finally, the sum of ARCH and GARCH parameters of TGARCH (p, q) models is almost equal to one that indicates the effects of shocks on rupee exchange rate against our benchmark economies is persistent for long run and tend to accumulate, which shows the volatility bunching. Hence, higher conditional volatility in the following figures 6 to 9 show that volatility continues to be high, when it is high, and low, when it is low. Finally, from asymmetric models we can say that negative shocks, in exchange rate of Pakistani rupee against discussed four economies are highly volatile, and highly persistent.

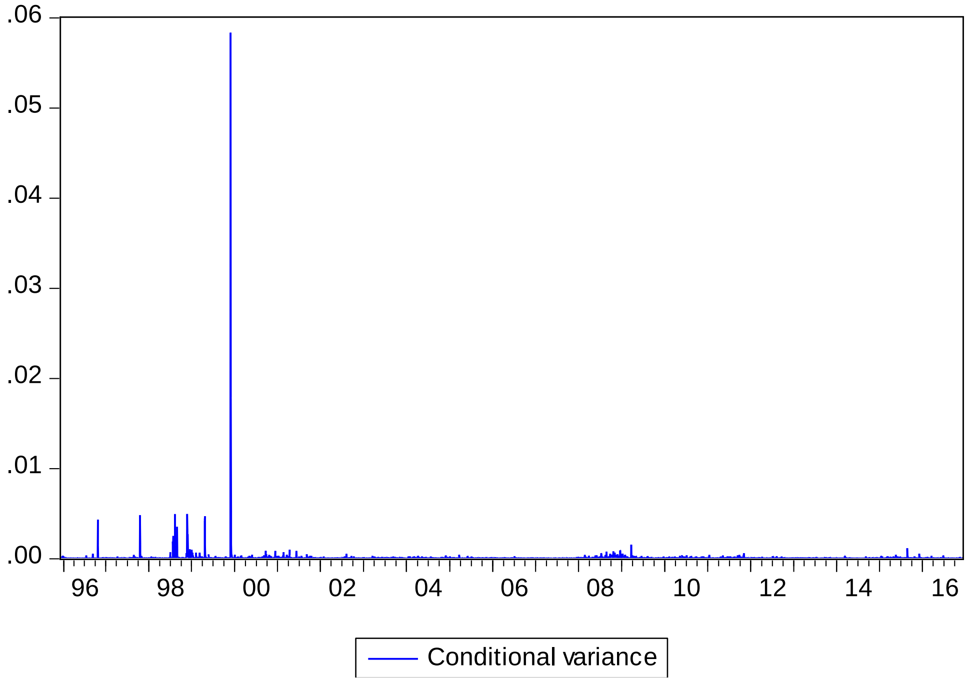


Figure 6: Conditional Variance Plot: PKR to Euro

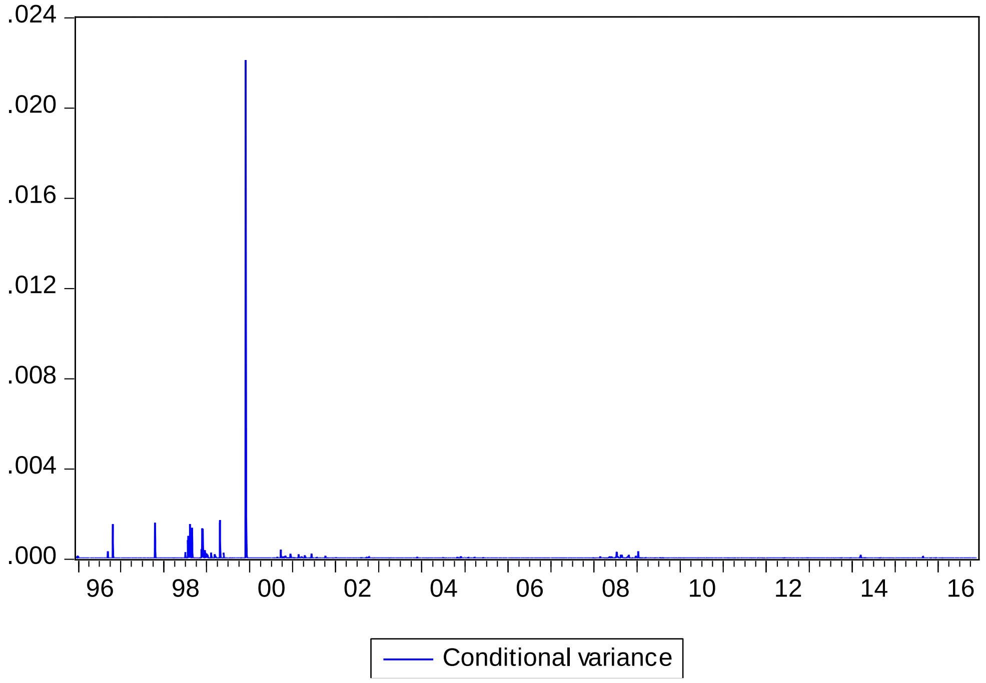


Figure 7: Conditional Variance Plot: PKR to US Dollar

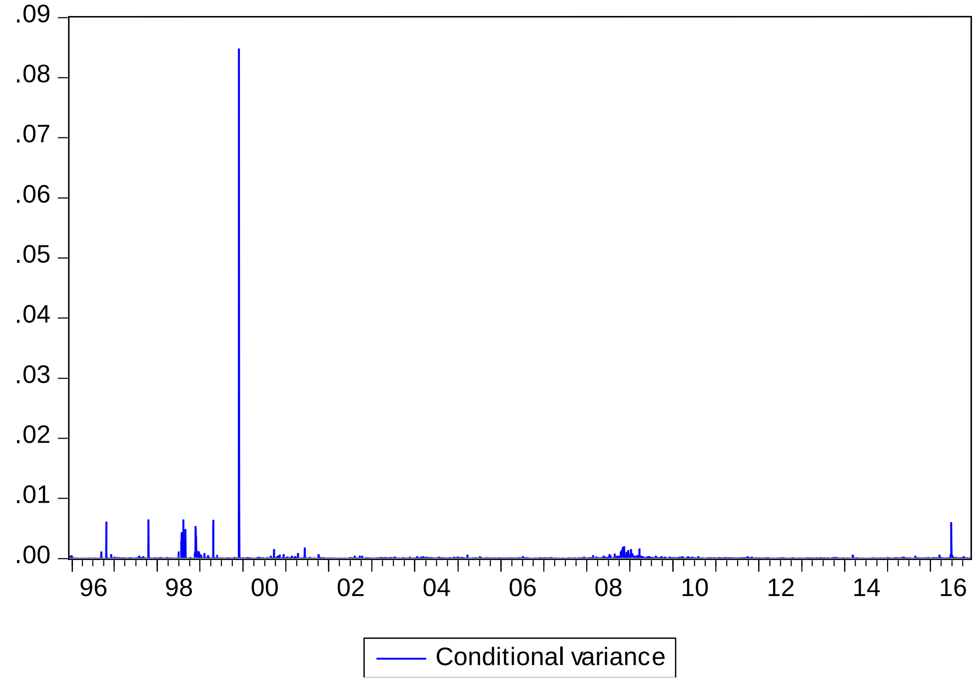


Figure 8: Conditional Variance Plot: PKR to GB Pound

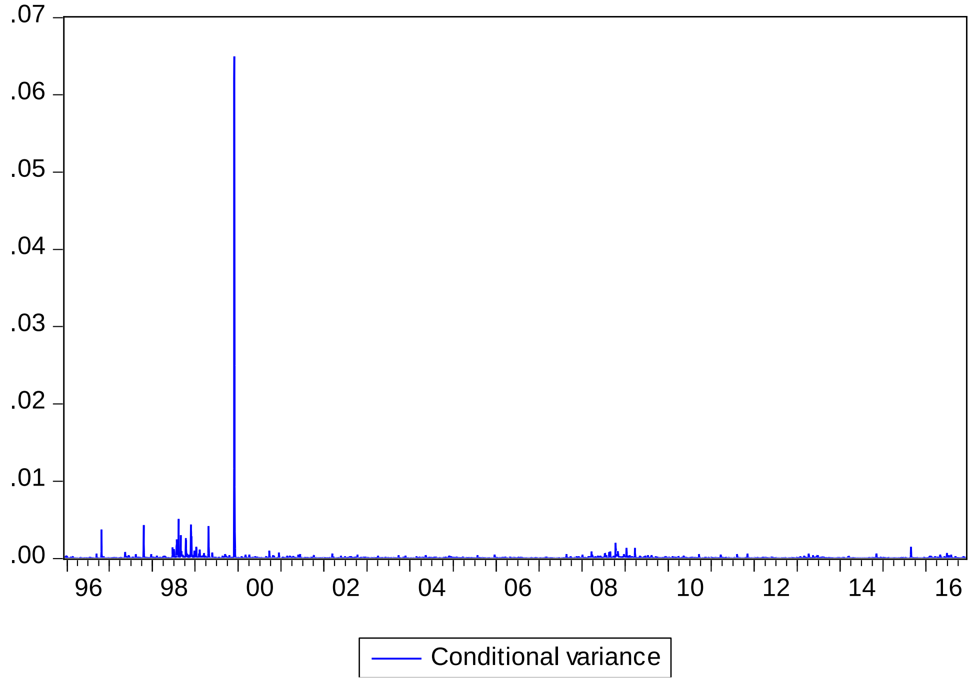


Figure 9: Conditional Variance Plot: PKR to Japanese yen

Bibliography

1. Ahmed E M, S. Z. (2011). Modeling Stock Market Volatility Using GARCH Models: Evidence From Sudan. *International Journal of Business and Social Science*, Vol. 2, No. 23, pp. 114-128.
2. 23. Bollerslev, T. C. (1992). (1992). ARCH modeling in finance: A review of the theory and empirical evidence . *Journal of econometrics*, 52(1-2), 5-59.
3. Ahmed E M, S. Z. (2011). Modeling Stock Market Volatility Using GARCH Models: Evidence From Sudan. *International Journal of Business and Social Science*, Vol. 2, No. 23, pp. 114-128.
4. D, N. (1991). *Conditional Heteroskedasticity in Asset Returns: A New Approach.* Econometrica, Vol. 59, No. 1, pp. 347-370. .
5. Dickey D A and Fuller W A. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of American Statistical Association Vol. 74, No. 366*, pp. 427-431.
6. Engel, R. (1982). Autoregressive Conditional Heteroskedasticity. *Econometrica, Vol. 50, No. 4*, 987-1007.
7. Engle, R. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, Vol. 50, No. 4, pp. 987-1007.
8. Glosten I R, Jagannathan R and Runkle . (1993). On the Relation Between the Expected Value and the Volatility of the Normal Excess Return on Stocks. *Journal of Finance Vol. 48, No. 5*, pp. 1779-1801.
9. Kyle, C. a. (1993).
10. Nelson, D. B. (1991). *Stationarity and persistence in the GARCH (l, l) model.* Econometric theory, 6(3), 318-334.

11. Phillips P C and Perron P. (1988). Testing for a Unit Root in Time Series Regression. *Biometrika, Vol. 75, No. 2,*, pp. 335-346.
12. Streissler, E. (2003). *Exchange Rates and International Finance Markets: An Asset-Theoretic Perspective with Schumpeterian Perspective.* (Vol. 5). Routledge.
13. Taylor, S. J. (1987). Forecasting the Volatility of Currency Exchange Rates.
14. Adjasi, Charles and Harvey, Simon K. and Agyapong, Daniel Akwesi, Effect of Exchange Rate Volatility on the Ghana Stock Exchange (December 28, 2008). African Journal of Accounting, Economics, Finance and Banking Research, Vol. 3, No. 3, December 2008. Available at SSRN: https://ssrn.com/abstract=1534178
15. Habib, Maurizio Michael and Mileva, Elitza and Stracca, Livio, The Real Exchange Rate and Economic Growth: Revisiting the Case Using External Instruments (June 17, 2016). ECB Working Paper No. 1921. Available at SSRN: https://ssrn.com/abstract=2797378
16. Smith, A. (1937). *The wealth of nations [1776]* (p. 421). na.
17. MacDonald, R. (1999). Exchange rate behaviour: are fundamentals important?. *The Economic Journal*, *109*(459), 673-691.
18. Mahmood, I., Ehsanullah, M., & Ahmed, H. (2011). Exchange rate volatility & macroeconomic variables in Pakistan. *Business Management Dynamics, 1 (2)*, 11-22.
19. Streissler, E. (2003). *Exchange Rates and International Finance Markets: An Asset-Theoretic Perspective with Schumpeterian Perspective* (Vol. 5). Routledge.
20. Habib, M. M., Mileva, E., & Stracca, L. (2016). The real exchange rate and economic growth: revisiting the case using external instruments.
21. Diebold, F. X., & Nerlove, M. (1989). The dynamics of exchange rate volatility: a multivariate latent factor ARCH model. *Journal of Applied econometrics*, *4*(1), 1-21.
22. Murari, K. (2015). Exchange Rate Volatility Estimation Using GARCH Models, with Special Reference to Indian Rupee Against World Currencies. *IUP Journal of Applied Finance*, *21*(1), 22.
23. Nelson, D. B. (1990). Stationarity and persistence in the GARCH (l, l) model. *Econometric theory*, *6*(3), 318-334.
24. Bougerol, P., & Picard, N. (1992). Stationarity of GARCH processes and of some nonnegative time series. *Journal of econometrics*, *52*(1-2), 115-127.
25. Kaur, H. (2004). Time varying volatility in the Indian stock market. *Vikalpa*, *29*(4), 25-42.

26. Abdalla, S. Z. S. (2012). Modelling exchange rate volatility using GARCH models: Empirical evidence from Arab countries. *International Journal of Economics and Finance*, *4*(3), 216.

27. Adjasi, C., Harvey, S. K., & Agyapong, D. A. (2008). Effect of exchange rate volatility on the Ghana Stock Exchange.

28. Mahmood, I., Ehsanullah, M., & Ahmed, H. (2011). Exchange rate volatility & macroeconomic variables in Pakistan. *Business Management Dynamics, 1 (2)*, 11-22.

29. Mandelbrot, B. (1963). New methods in statistical economics. *Journal of political economy*, *71*(5), 421-440.

30. Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues.